

## HW 8 , Math 530, Fall 2014

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- QUESTION 1.** (i) Show that the groups  $A_5$  and  $S_5$  each have 10 subgroups of size 3 and 6 subgroups of size 5. [Hint: Note that  $A_n$  is a simple group for every  $n \geq 5$ . Also ask yourself this question: If  $\alpha \in S_n$  and of odd prime order, where does  $\alpha$  "live"? ]
- (ii) Show that every group of order 45 is abelian.
- (iii) Let  $G$  be a group of order 12. Show that  $G$  must have a normal 2-Sylow subgroup (i.e., of order 4) or 3-Sylow subgroup.
- (iv) Let  $G$  be a group of order 70. Show that  $G$  has a normal subgroup of order 35. Show that all elements of order 2 in  $G$  are conjugate to each other.

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